STRUCTURAL MECHANICS

LEARNING OBJECTIVE

CHAPTER-

1. **Properties of Materials** Classification of materials, elastic materials, plastic materials, ductile materials, brittle materials. Tensile test, compressive test, impact test, fatigue test, torsion test.
2. **Simple Stresses and Strains** Concept of stress, normal and shear stresses due to torsion Concept of strain, strain and deformation, longitudinal and lateral strain, poison's ratio, Volumetric strain Hooke's law, modulli of elasticity and rigidity, Bulk modulus of elasticity, relationship between the elastic constants. Stresses and strains in bars subjected to tension and compression. Extension of uniform bar under its own weight, stress produces in compound bars (two or three) due to axial load. Stressstrain diagram for mild steel, mechanical properties, factor of safety
   Temperature stresses and strains
3. **Bending Moment and Shear Force** Concept of a beam and supports (Hinges, Roller and Fixed), types of beams: simply supported, cantilever, fixed and continuous beams
   Types of loads (point, uniformly distributed and varying loads) Concept of bending moment and shear force, sign conventions Bending Moment and shear force diagrams for cantilever, simply supported and over hanging beams subjected to concentrated, uniformly distributed and uniformly varying loads (B.M. and S.F. diagrams should preferably be drawn on graph paper. Relationship between load, shear force and bending moment, point of maximum bending moment and contraflexure.
4. **Second Moment of Area** Concept of second moment of area, radius of gyration
   Theorems of parallel and perpendicular axes Second moment of area for sections of Rectangle, Triangle, Circle, Trapezium, Angle, Tee,
   I, Channel and Compound sections. (No derivation)
5. **Bending and Shear Stresses** Theory of simple bending Application of the equation M / I= sigma / Y= E/R (No derivation is required) Moment of resistance, sectional modulus and permissible bending stresses in circular, rectangular, I, T and L sections; Comparision of strengths of the above sections.
6. **Slope and Deflection** Necessity for determination of reflection Moment area theorems (no derivation) Computation of slopes and deflections using moment area theorems for: (a) Simple supported beam with UDL over entire span and concentrated load at any point
   (c) Cantilever with UDL over entire span and concentrated load at free end
7. **Columns Theory of columns**, Euler, Rankine's and I.S. formulae. Combined Direct and Bending Stresses Concentric and eccentric loads, eccentricity Effect of eccentric load on the section, stresses due to eccentric loads, examples in the case of short columns. Effect of wind pressure on walls and chimneys; water pressure on dams and earth pressure on retaining walls their causes of failures and their stability.
8. **Analysis of Trusses** Concept of a frame, redundant and deficient frame, End supports, ideal and practical trusses. Analysis of trusses by: (i) Methods of joints (ii) Method of sections and
   (iii) Graphical method

CHAPTER-1 PROPERTIES OF MATERIAL

- Elasticity: Ability of a body to resist a distorting influence or stress and to return to its original size and shape when the stress is removed.
- Plasticity: Ability of a material to undergo irreversible or permanent deformations without breaking or rupturing; opposite of brittleness.
- Malleability: Ability of the material to be flattened into thin sheets under applications of heavy compressive forces without cracking by hot or cold working means.
- Ductility: Ability of a material to deform under tensile load (% elongation).
- Flexibility: Ability of an object to bend or deform in response to an applied force; pliability; complementary to stiffness.
- Toughness: Ability of a material to absorb energy (or withstand shock) and plastically deform without fracturing (or rupturing); a material's resistance to fracture when stressed; combination of strength and plasticity
- Brittleness: Ability of a material to break or shatter without significant deformation when under stress; opposite of plasticity,examples:glass,concrete,cast iron,ceramics etc.

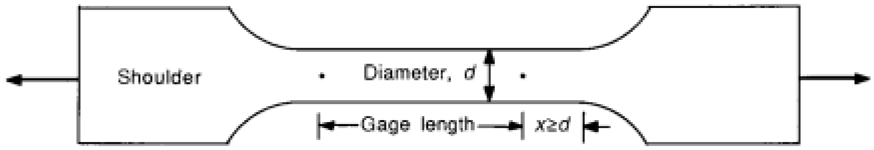
TEST OF MATERIALS

1.TENSILE TEST-

TENSILE TESTS are performed for several reasons. The results of tensile tests are used in selecting materials for engineering applications. Tensile properties frequently are included in material specifications to ensure quality. Tensile properties often are measured during development of new materials and processes, so that different materials and processes can be compared. Finally, tensile properties often are used to predict the behavior of a material under forms of loading other than uniaxial tension. The strength of a material often is the primary concern. The strength of interest may be measured in terms of either the stress necessary to cause appreciable plastic deformation or the maximum stress that the material can withstand. These measures of strength are used, with appropriate caution (in the form of safety factors), in engineering design. Also of interest is the material's ductility, which is a measure of how much it can be deformed before it fractures. Rarely is ductility incorporated directly in design; rather, it is included in material specifications to ensure quality and toughness. Low ductility in a tensile test often is accompanied by low resistance to fracture under other forms of loading. Elastic properties also may be of interest, but special techniques must be used to measure these properties during tensile testing, and more accurate measurements can be made by ultrasonic techniques.

Tensile Specimens and Testing Machines

Tensile Specimens- Consider the typical tensile specimen . It has enlarged ends or shoulders for gripping. The important part of the specimen is the gage section. The cross-sectional area of the gage section is reduced relative to that of the remainder of the specimen so that deformation and failure will be localized in this region. The gage length is the region over which measurements are made and is centered within the reduced section. The distances between the ends of the gage section and the shoulders should be great enough so that the larger ends do not constrain deformation within the gage section, and the gage length should be great relative to its diameter. Otherwise, the stress state will be more complex than simple tension. Detailed descriptions of standard specimen shapes are given and in subsequent chapters on tensile testing of specific materials. There are various ways of gripping the specimen, some of which are illustrated .The end may be screwed into a threaded grip, or it may be pinned; butt ends may be used, or the grip section may be held between wedges. There are still other methods .The most important concern in the selection of a gripping method is to ensure that the specimen can be held at the maximum load without slippage or failure in the grip section. Bending should be minimized.



2-COMPRESSIVE TEST

Compressive strength or **compression strength** is the capacity of a material or structure to withstand loads tending to reduce size, as opposed to tensile strength, which withstands loads tending to elongate. In other words, compressive strength resists compression (being pushed together), whereas tensile strength resists tension (being pulled apart). In the study of strength of materials, tensile strength, compressive strength, and shear strength can be analyzed independently.

Some materials fracture at their compressive strength limit; others deform irreversibly, so a given amount of deformation may be considered as the limit for compressive load. Compressive strength is a key value for design of structures.

3-TORSION TEST

A torsion test measures the strength of any material against maximum twisting forces. Itis an extremely common test used in material mechanics to measure how much of a twist acertain material can withstand before cracking or breaking. This applied pressure is referred to astorque. Materials typically used in the manufacturing industry, such as metal fasteners andbeams, are often subject to torsion testing to determine their strength under duress.There are three broad categories under which a torsion test can take place: failure testing, prooftesting and operational testing. Failure testing involves twisting the material until it breaks. Prooftesting observes whether a material can bear a certain amount of torque load over a given periodof time. Operational testing tests specific products to confirm their elastic limit before going onthe market.It is critical for the results of each torsion test to be recorded. Recording is done through creatinga stress-strain diagram with the angle of twist values on the X-axis and the torque values on theY-axis. Using a torsion testing apparatus, twisting is performed at quarter-degree increments withthe torque that it can withstand recorded. The strain corresponds to the twist angle, and the stresscorresponds to the torque measured.

4-IMPACT TEST

The **Charpy impact test**, also known as the **Charpy V-notch test**, is a standardized high strain-rate test which determines the amount of energy absorbed by a material during fracture. This absorbed energy is a measure of a given material's notch toughness and acts as a tool to study temperature-dependent ductile-brittle transition. It is widely applied in industry, since it is easy to prepare and conduct and results can be obtained quickly and cheaply. A disadvantage is that some results are only comparative.

The Test was developed around 1900 by S.B. Russell (1898, American) and Georges Charpy (1901, French).The test became known as the Charpy test In the early 1900s due to the technical contributions and standardization efforts by Charpy. The test was pivotal in understanding the fracture problems of ships during World War II.

Today it is utilized in many industries for testing materials, for example the construction of pressure vessels and bridges to determine how storms will affect the materials used.

he apparatus consists of a pendulum of known mass and length that is dropped from a known height to impact a notched specimen of material. The energy transferred to the material can be inferred by comparing the difference in the height of the hammer before and after the fracture (energy absorbed by the fracture event).

The notch in the sample affects the results of the impact test, thus it is necessary for the notch to be of regular dimensions and geometry. The size of the sample can also affect results, since the dimensions determine whether or not the material is in plane strain. This difference can greatly affect the conclusions made.

CHAPTER-2 SIMPLE STRESS AND STRAIN BEHAVIOUR OF MATERIALS

1. Introduction

When a force is applied on a body it suffers a change in shape, that is, it deforms. A force to resist the deformation is also set up simultaneously within the body and it increases as the deformation continues. The process of deformation stops when the internal resisting force equals the externally applied force. If the body is unable to put up full resistance to external action, the process of deformation continues until failure takes place. The deformation of a body under external action and accompanying resistance to deform are referred to by the terms strain and stress respectively.

2. Stresses

Stress is defined as the internal resistance set up by a body when it is deformed. It is measured in N/m^2 and this unit is specifically called Pascal (Pa). A bigger unit of stress is the mega Pascal (MPa).

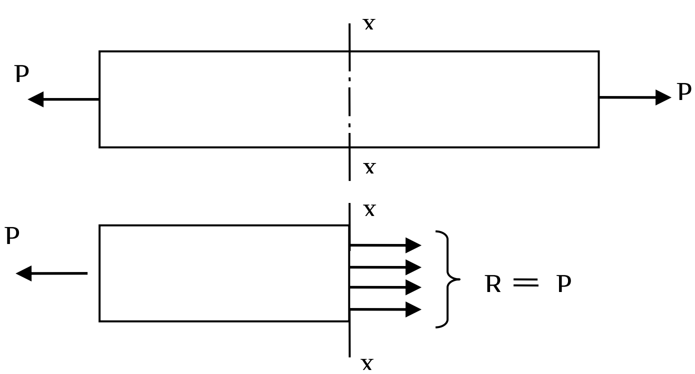
1 Pa = $1N/m^2$,

1MPa = 10^6 N/m^2 = $1N/mm^2$.

2.1. Three Basic Types of Stresses

Basically three different types of stresses can be identified. These are related to the nature of the deforming force applied on the body. That is, whether they are tensile, compressive or shearing.

2.1.1. Tensile Stress



Consider a uniform bar of cross sectional area A subjected to an axial tensile force P. The stress at any section x-x normal to the line of action of the tensile force P is specifically called tensile stress p_t . Since internal resistance R at x-x is equal to the applied force P, we have,

p_t = (internal resistance at x-x)/(resisting area at x-x)

=R/A

=P/A.

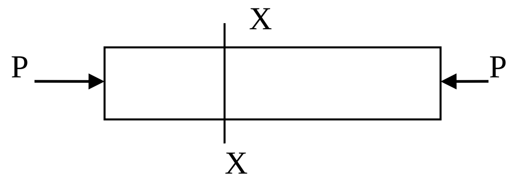
Under tensile stress the bar suffers stretching or elongation.

2.1.2. Compressive Stress

If the bar is subjected to axial compression instead of axial tension, the stress developed at x-x is specifically called compressive stress p_c.

p_c =R/A

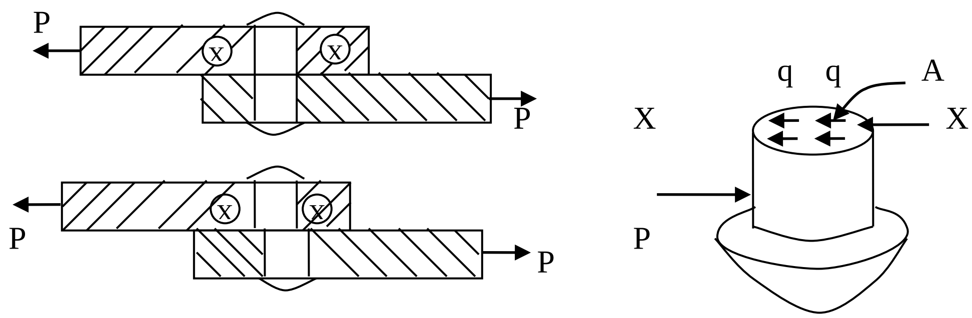
= P/A.



Under compressive stress the bar suffers shortening.

2.1.3. Shear Stress

Consider the section x-x of the rivet forming joint between two plates subjected to a tensile force P as shown in figure.



The stresses set up at the section x-x acts along the surface of the section, that is, along a direction tangential to the section. It is specifically called shear or tangential stress at the section and is denoted by q.

$$q = R/A$$
$$= P/A.$$

2.2. Normal or Direct Stresses

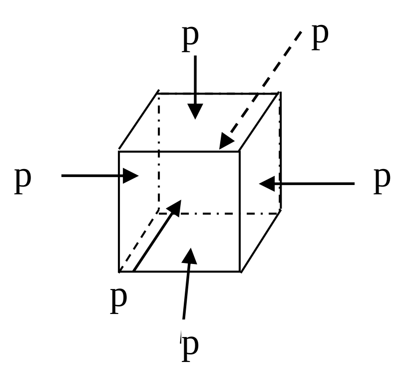
When the stress acts at a section or normal to the plane of the section, it is called a normal stress or a direct stress. It is a term used to mean both the tensile stress and the compressive stress.

2.3. Simple and Pure Stresses

The three basic types of stresses are tensile, compressive and shear stresses. The stress developed in a body is said to be simple tension, simple compression and simple shear when the stress induced in the body is (a) single and (b) uniform. If the condition (a) alone is satisfied, the stress is called pure tension or pure compression or pure shear, as the case may be.

2.4. Volumetric Stress

Three mutually perpendicular like direct stresses of same intensity produced in a body constitute a volumetric stress. For example consider a body in the shape of a cube subjected equal normal pushes on all its six faces. It is now subjected to equal compressive stresses p in all the three mutually perpendicular directions. The body is now said to be subjected to a volumetric compressive stress p.



Volumetric stress produces a change in volume of the body without producing any distortion to the shape of the body.

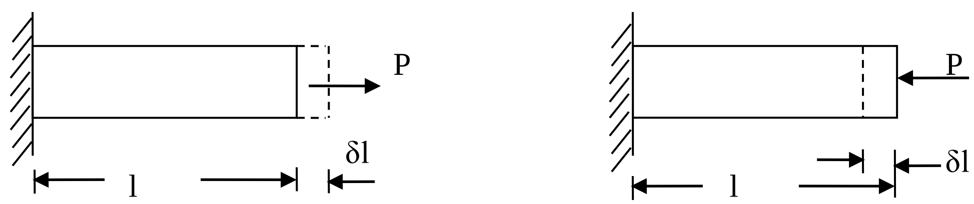
3. Strains

Strain is defined a the ratio of change in dimension to original dimension of a body when it is deformed. It is a dimensionless quantity as it is a ratio between two quantities of same dimension.

3.1. Linear Strain

Linear strain of a deformed body is defined as the ratio of the change in length of the body due to the deformation to its original length in the direction of the force. If l is the original length and δl the change in length occurred due to the deformation, the linear strain e induced is given by

$$\mathbf{e = \delta l / l}.$$



Linear strain may be a tensile strain , $\mathbf{e_t}$ or a compressive strain $\mathbf{e_c}$ according as δl refers to an increase in length or a decrease in length of the body. If we consider one of these as +ve then the other should be considered as –ve, as these are opposite in nature.

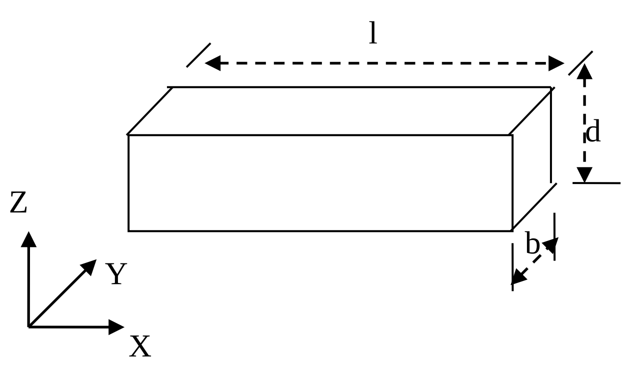
3.2.Lateral Strain

Lateral strain of a deformed body is defined as the ratio of the change in length (breadth of a rectangular bar or diameter of a circular bar) of the body due to the deformation to its original length (breadth of a rectangular bar or diameter of a circular bar) in the direction perpendicular to the force.

3.3. Volumetric Strain

Volumetric strain of a deformed body is defined as the ratio of the change in volume of the body to the deformation to its original volume. If V is the original volume and δV the change in volume occurred due to the deformation, the volumetric strain e_v induced is given by $\mathbf{e_v = \delta V/V}$

Consider a uniform rectangular bar of length l, breadth b and depth d as shown in figure. Its volume V is given by,



$V = lbd$

$\delta V = \delta l\, bd + \delta b\, ld + \delta d\, lb$

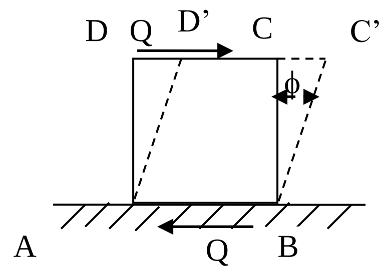
$\delta V / V = (\delta l / l) + (\delta b / b) + (\delta d / d)$

$e_v = e_x + e_y + e_z$

This means that volumetric strain of a deformed body is the sum of the linear strains in three mutually perpendicular directions.

3.4. Shear Strain

Shear strain is defined as the strain accompanying a shearing action. It is the angle in radian measure through which the body gets distorted when subjected to an external shearing action. It is denoted by Φ.



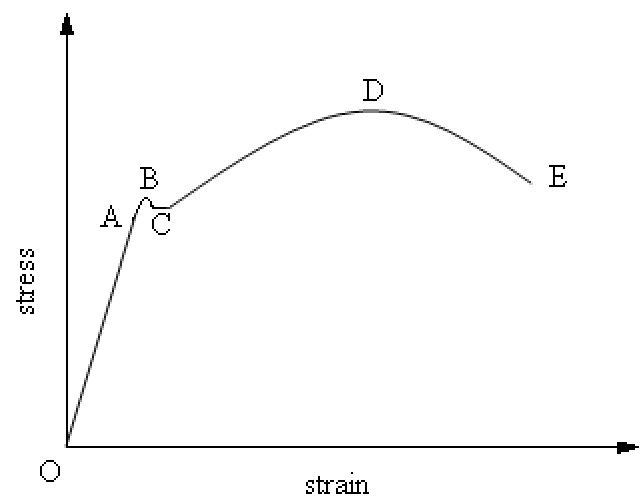
Consider a cube ABCD subjected to equal and opposite forces Q across the top and bottom forces AB and CD. If the bottom face is taken fixed, the cube gets distorted through angle ϕ to the shape ABC’D’. Now strain or deformation per unit length is

Shear strain of cube = CC’ / CD = CC’ / BC = ϕ radian

4. Relationship between Stress and Strain

Relationship between Stress and Strain are derived on the basis of the elastic behaviour of material bodies.

A standard mild steel specimen is subjected to a gradually increasing pull by Universal Testing Machine. The stress-strain curve obtained is as shown below.



A	-Elastic Limit
B	- Upper Yield Stress
C	- Lower Yield Stress
D	-Ultimate Stress
E	-Breaking Stress

4.1. Elasticity and Elastic Limit

Elasticity of a body is the property of the body by virtue of which the body regains its original size and shape when the deformation force is removed. Most materials are elastic in nature to a lesser or greater extend, even though perfectly elastic materials are very rare.

The maximum stress upto which a material can exhibit the property of elasticity is called the elastic limit. If the deformation forces applied causes the stress in the material to exceed the elastic limit, there will be a permanent set in it. That is the body will not regain its original shape and size even after the removal of the deforming force completely. There will be some residual strain left in it.

Yield stress

When a specimen is loaded beyond the elastic limit the stress increases and reach a point at which the material starts yielding this stress is called yield stress.

Ultimate stress

Ultimate load is defined as maximum load which can be placed prior to the breaking of the specimen. Stress corresponding to the ultimate load is known as ultimate stress.

Working stress

Working stress= Yield stress/Factor of safety.

4.2. Hooke's Law

Hooke's law states that stress is proportional to strain upto elastic limit. If I is the stress induced in a material and e the corresponding strain, then according to Hooke's law,

$p / e = E$, a constant.

This constant E is called the modulus of elasticity or Young's Modulus, (named after the English scientist Thomas Young).

It has later been established that Hooke's law is valid only upto a stress called the limit of proportionality which is slightly less than the elastic limit.

4.3. Elastic Constants

Elastic constants are used to express the relationship between stresses and strains. Hooke's law , is stress/strain = a constant, within a certain limit. This means that any stress/corresponding strain = a constant, within certain limit. It follows that there can be three different types of such constants. (which we may call the elastic constants or elastic modulae) corresponding to three distinct types of stresses and strains. These are given below.

(i)Modulus of Elasticity or Young's Modulus (E)

Modulus of Elasticity is the ratio of direct stress to corresponding linear strain within elastic limit. If p is any direct stress below the elastic limit and e the corresponding linear strain, then $\mathbf{E = p / e}$.

(ii)Modulus of Rigidity or Shear Modulus (G)

Modulus of Rigidity is the ratio of shear stress to shear strain within elastic limit. It is denoted by N,C or G. if q is the shear stress within elastic limit and ϕ the corresponding shear strain, then $\mathbf{G = q / \phi}$.

(iii) Bulk Modulus (K)

Bulk Modulus is the ratio of volumetric stress to volumetric strain within the elastic limit. If p_v is the volumetric stress within elastic limit and e_v the corresponding volumetric strain, we have $K = p_v / e_v$.

5. Poisson's Ratio

Any direct stress is accompanied by a strain in its own direction and called linear strain and an opposite kind strain in every direction at right angles to it, lateral strain. This lateral strain bears a constant ratio with the linear strain. This ratio is called the Poisson's ratio and is denoted by (1/m) or μ.

Poisson's Ratio = Lateral Strain / Linear Strain.

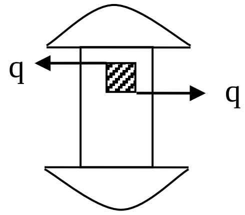
Value of the Poisson's ratio for most materials lies between 0.25 and 0.33.

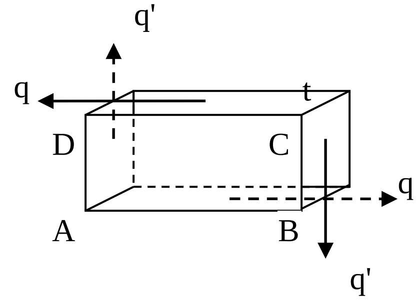
6. Complementary Strain

Consider a rectangular element ABCD of a body subjected to simple shear of intensity q as shown. Let t be the thickness of the element.

Total force on face AB is , F_{AB} = stress X area = q X AB X t.

Total force on face CD is, F_{CD} = q X CD Xt = q XAB Xt.





FAB and FCD being equal and opposite, constitute a couple whose moment is given by,

M =F_{AB} X BC = q XAB X BC X t

Since the element is in equilibrium within the body, there must be a balancing couple which can be formed only by another shear stress of some intensity q' on the faces BC and DA. This shear stress is called the complementary stress.

F_{BC} = q' X BC X t

F_{DA} = q' X DA X t = q' X BC Xt

The couple formed by these two forces is M' = F_{BC} X AB = q' X BC X t

For equilibrium, M' = M.

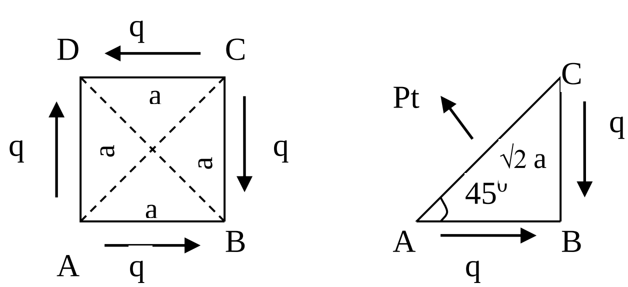
Therefore q' = q

This enables us to make the following statement.

In a state of simple shear a shear stress of any intensity along a plane is always accompanied by a complementary shear stress of same intensity along a plane at right angles to the plane.

7. Direct Stresses Developed Due to Simple Shear.

Consider a square element of side a and thickness t in a state of simple shear as shown in figure. It is clear that the shear stress on the forces of element causes it to elongate in the direction of the diagonal BD. Therefore a tensile stress of same intensity pt is induced in the elements along BD. ie, across the plane of the diagonal AC. The triangular portion ABC of the element is in equilibrium under the action of the following.



(1)F_{AC} = Normal force on face AC = pt X AC X t = pt X √2 aXt

(2)F_{AB} = Tangential force on face AB = q X BC X t = q aXt

(3)FBC = Tangential force on face BC = q X BC X t = q aXt

For equilibrium in the direction normal to AC,

FAC – F_{AB} cos45 – F_{BC} cos45 = 0

Pt X √2 at – q at X 1/√2 - q at X 1/√2 = 0

√2 pt – 2 q /√2 = 0

pt = q

It can also be seen that the shear stress on the faces of the element causes it to foreshorten in the direction of the diagonal BD. Therefore a compressive stress pc is induced in the element in the direction AC, ie across the plane of the diagonal BD. It can also be shown that pc = q.

It can thus be concluded that simple shear of any intensity gives rise to direct stresses of same intensity along the two planes inclined at 45º to the shearing plane. The stress along one of these planes being tensile and that along the other being compressive.

8. Relationship among the elastic constants

8.1. Relationship between modulus of elasticity and modulus of rigidity

Consider a square element ABCD of side ‘a’ subjected to simple shear of intensity q as shown in figure.

It is deformed to the shape AB’C’D under the shear stress. Drop perpendicular BE to the diagonal DB’. Let Φ be the shear strain induced and let N be the modulus of rigidity.

The diagonal DB gets elongated to DB’. Hence there is tensile strain et in the diagonal.

et = (DB’ – DB) / DB = EB’ / DB

since this deformation is very small we can take L BB’E = 45º

EB’ = BB’ / $\sqrt{2}$ = AB tan Φ / $\sqrt{2}$ = a tan Φ / $\sqrt{2}$

DB = $\sqrt{2}$ a

et = (a tan Φ / $\sqrt{2}$)/ $\sqrt{2}$ a = tan Φ /2 = = Φ / 2 since Φ is small

ie et = ½ X q/N ------------------ (1)

We know that stress along the diagonal DB is a pure tensile stress pt = q and that along the diagonal AC is a pure compressive stress pc also equal to q. hence the strain along the diagonal DB is et = q/E + 1/m X q/E

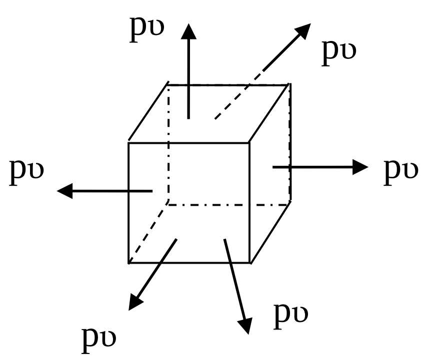
Ie et = q/E (1+1/m) ------------------ (2)

From (1) and (2) we have,

E = 2N(1+1/m)

This is the required relationship between E and N.

8.2. Relationship between Modulus of Elasticity E and Bulk Modulus K



Consider a cube element subjected to volumetric tensile stress pυ in X,Y and Z directions. Stress in each direction is equal to pυ. ie px = py = pz =pυ

Consider strains induced in X-direction by these stresses. px induces tensile strain , while py and pz induces compressive strains. Therefore,

ex = px/E – 1/m[py/E + pz/E] = pυ/E[1-2/m]

due to the perfect symmetry in geometry and stresses

ey = pυ/E[1-2/m]

ez = pυ/E[1-2/m]

K = p υ / e υ =p υ/(ex+ey+ez) = pυ/[3pυ/E(1-2/m)]

ie E = 3K(1-2/m) is the required relationship.

8.3. Relationship among the constants

From above,

E = 2N[1+(1/m)] and E = 3K[1-(2/m)]

E = 3K[1-2(E/2N -1)] = 3K[1-E/N +2]
9K = E[1+(3K/N)] = E[(N+3K)/N]
E = 9NK/(N+3K)

9. Bars of uniform section

Consider a bar of length l and Cross sectional area A. Let P be the axial pull on the bar,p the stresss induced ,e the strain in the bar and δl is the elongation.

Then p= P/A

e= p/E =P/(AE) -------------(1)

e= δl/l -------------(2)

equating (1) and (2)

δl = Pl / (AE)

CHAPTER-3 BENDING MOMENT AND SHEAR FORCE

TYPES OF LOADS

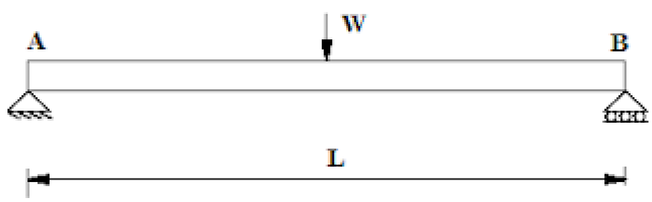
A beam is usually horizontal member and load which will be acting over the beam will be usually vertical loads. There are following types of loads as mentioned here and we will discuss each type of load in detail.

- Point load or concentrated load
- Uniformly distributed load
- Uniformly varying load

- **Point load or concentrated load**

  Point load or concentrated load, as name suggest, acts at a point on the beam. If we will see practically, point load or concentrated load also distributed over a small area but we can consider such type of loading as point loading and hence such type of load could be considered as point load or concentrated load.

  Following figure displayed here indicates the beam AB of length L which will be loaded with point load W at the midpoint of the beam. Load W will be considered here as the point load.

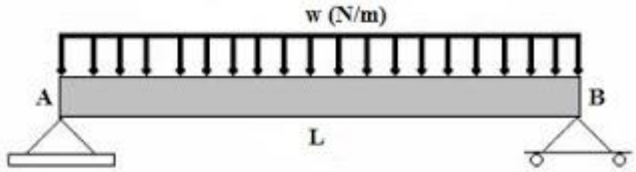


- **Uniformly distributed load**

  Uniformly distributed load is the load which will be distributed over the length of the beam in such a way that rate of loading will be uniform throughout the distribution length of the beam.

  Uniformly distributed load is also expressed as U.D.L and with value as w N/m. During determination of the total load, total uniformly distributed load will be converted in to point load by multiplying the rate of loading i.e. w (N/m) with the span of load distribution i.e. L and will be acting over the midpoint of the length of the uniformly load distribution.

  Let us consider the following figure, a beam AB of length L is loaded with uniformly distributed load and rate of loading is w (N/m).



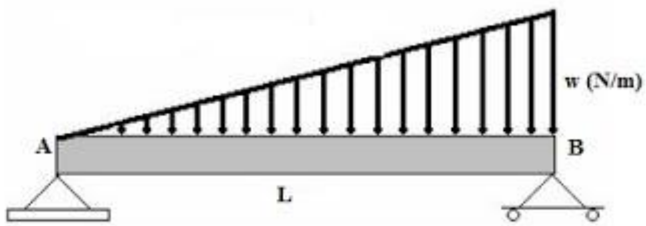
Total uniformly distributed load, P = w*L

- **Uniformly varying load**

  Uniformly varying load is the load which will be distributed over the length of the beam in such a way that rate of loading will not be uniform but also vary from point to point throughout the distribution length of the beam.

  Uniformly varying load is also termed as triangular load. Let us see the following figure, a beam AB of length L is loaded with uniformly varying load.

  We can see from figure that load is zero at one end and increases uniformly to the other end. During determination of the total load, we will determine the area of the triangle and the result i.e. area of the triangle will be total load and this total load will be assumed to act at the C.G of the triangle.



Total load, P = w*L/2

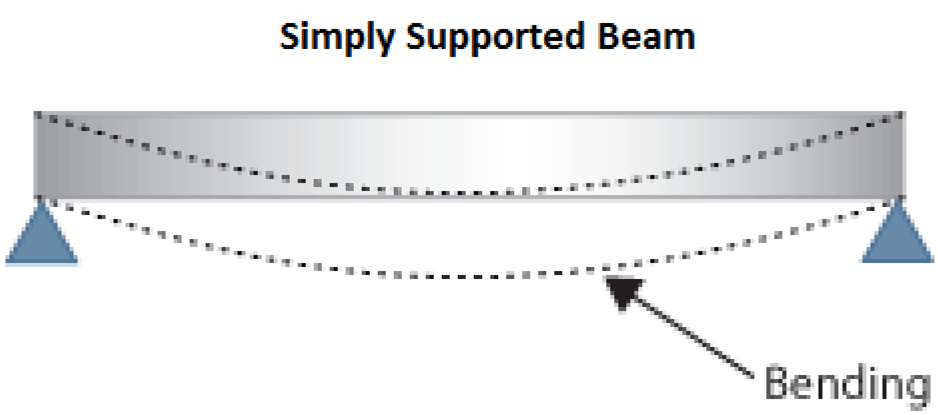
<u>TYPES OF BEAMS</u>

The four different types of beams are:

1. Simply Supported Beam
2. Fixed Beam
3. Cantilever Beam
4. Continuously Supported Beam

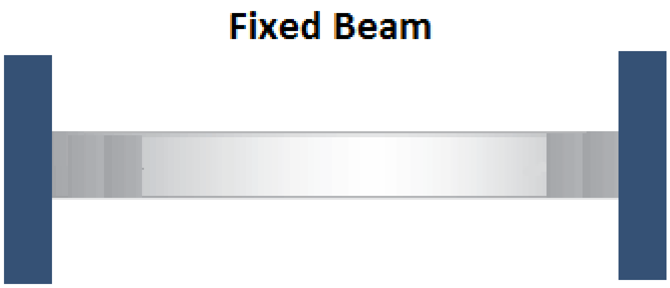
1. Simply Supported Beam

If the ends of a beam are made to rest freely on supports beam, it is called a simple (freely) supported beam.



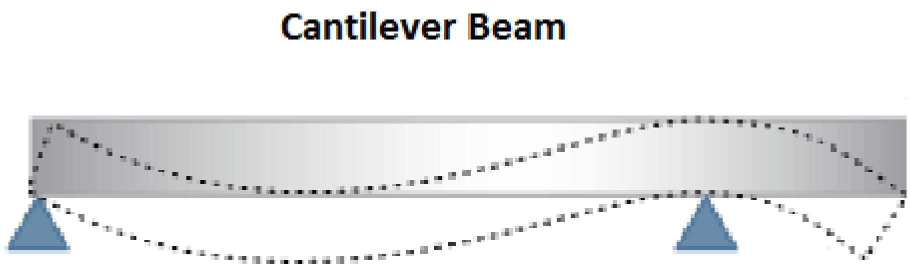
2. Fixed Beam

If a beam is fixed at both ends it is free called fixed beam. Its another name is a built-in beam.



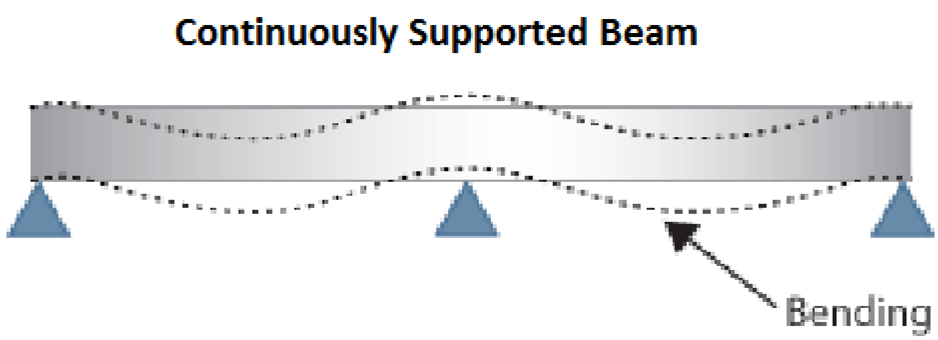
3. Cantilever Beam

If a beam is fixed at one end while the other end is free, it is called cantilever beam.



4. Continuously Supported Beam

If more than two supports are provided to the beam, it is called continuously supported beam.

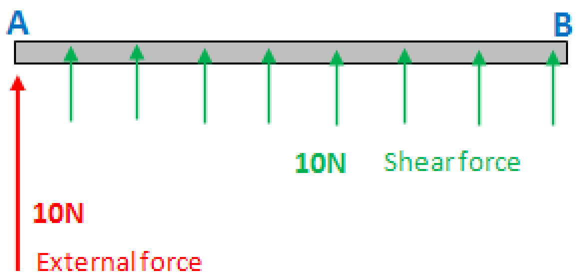


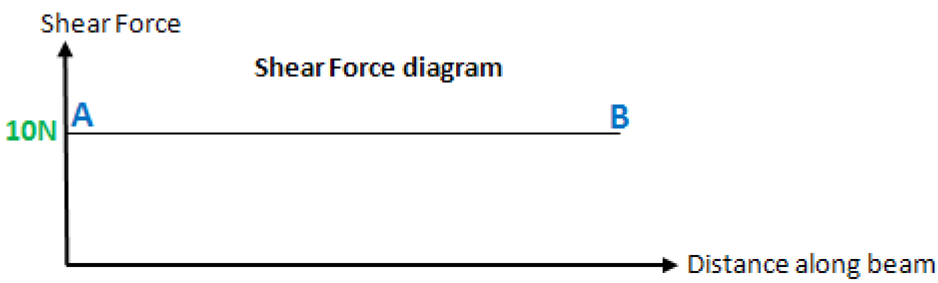
<u>TYPES OF SUPPORT</u>

Different types of external supports are as follows:

- Fixed support
- Pinned support or hinged support
- Roller support
- Link support
- Simple support

Below a force of 10N is exerted at point A on a beam. This is an external force. However because the beam is a rigid structure,the force will be internally transferred all along the beam. This internal force is known as shear force. The shear force between point A and B is usually plotted on a shear force diagram. As the shear force is 10N all along the beam, the plot is just a straight line, in this example.



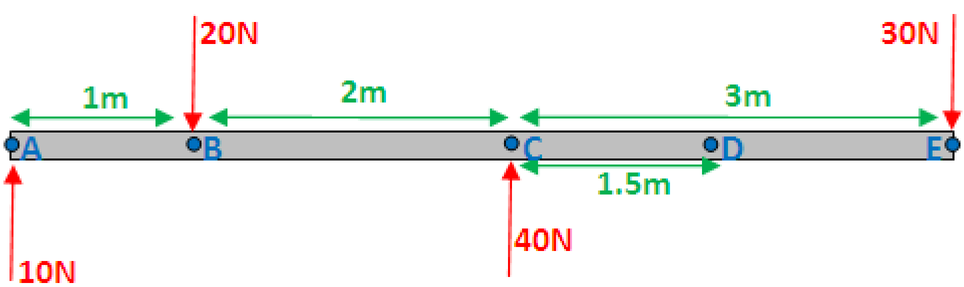


The idea of shear force might seem odd, maybe this example will help clarify. Imagine pushing an object along a kitchen table, with a 10N force. Even though you're applying the force only at one point on the object, it's not just that point of the object that moves forward. The whole object moves forward, which tells you that the force must have transferred all along the object, such that every atom of the object is experiencing this 10N force.

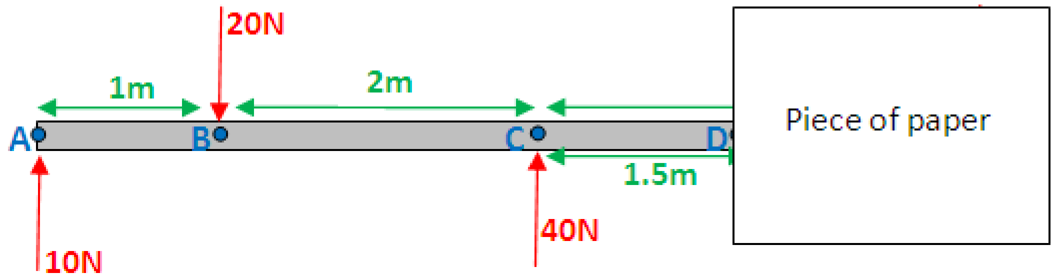
Please note that this is not the full explanation of what shear force actually is.

Basic shear diagram

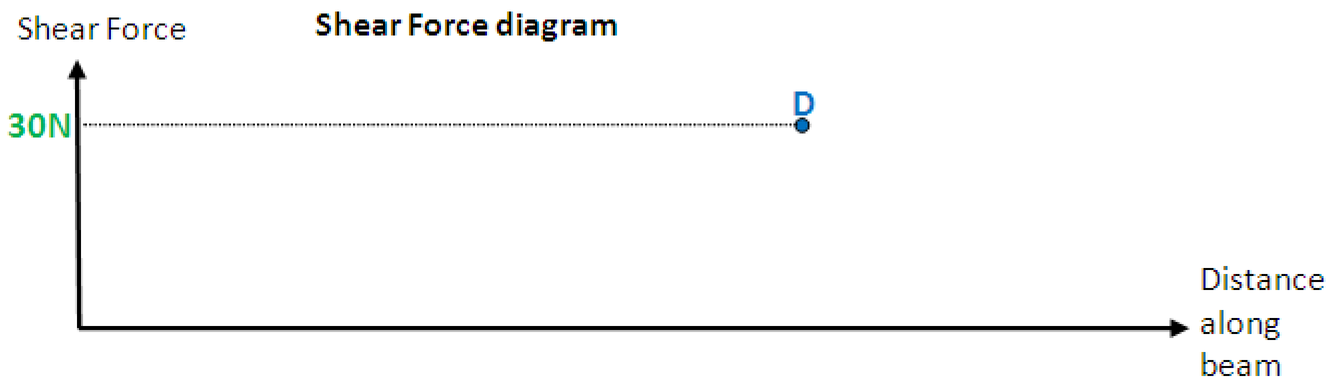
What if there is more than one force, as shown in the diagram below, what would the shear force diagram look like then?



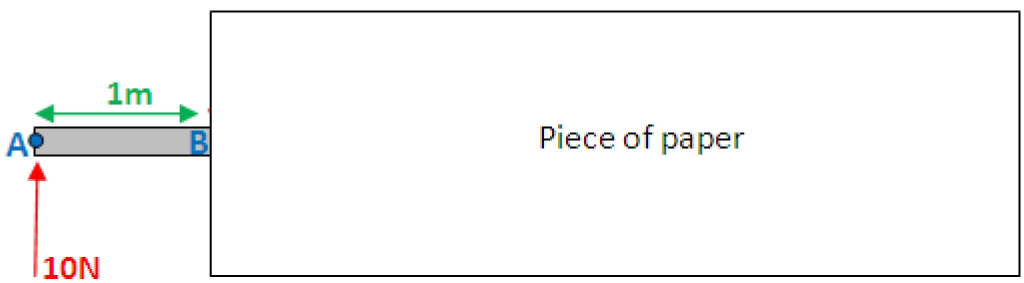
The way you go about this is by figuring out the shear force at points A,B,C,E (as there is an external force acting at these points). The way you work out the shear force at any point, is by covering (either with your hand or a piece of paper), everything to right of that point, and simply adding up the external forces. Then plot the point on the shear force diagram. For illustration purposes, this is done for point D:



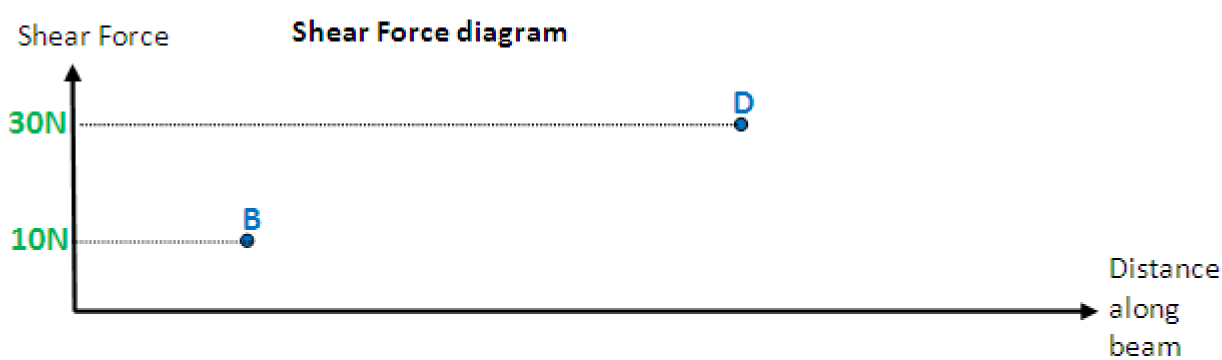
Shear force at D = 10N - 20N + 40N = 30Newtons



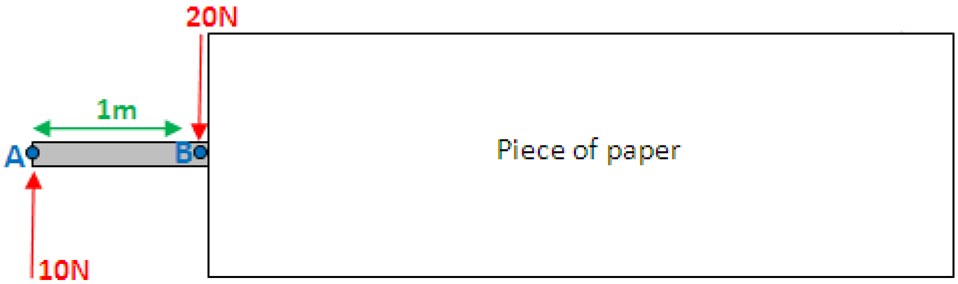
Now, let's do this for point B. BUT - slight complication - there's a force acting at point B, are you going to include it? The answer is both yes and no. You need to take 2 measurements. Firstly put your piece of paper, so it's JUST before point B:



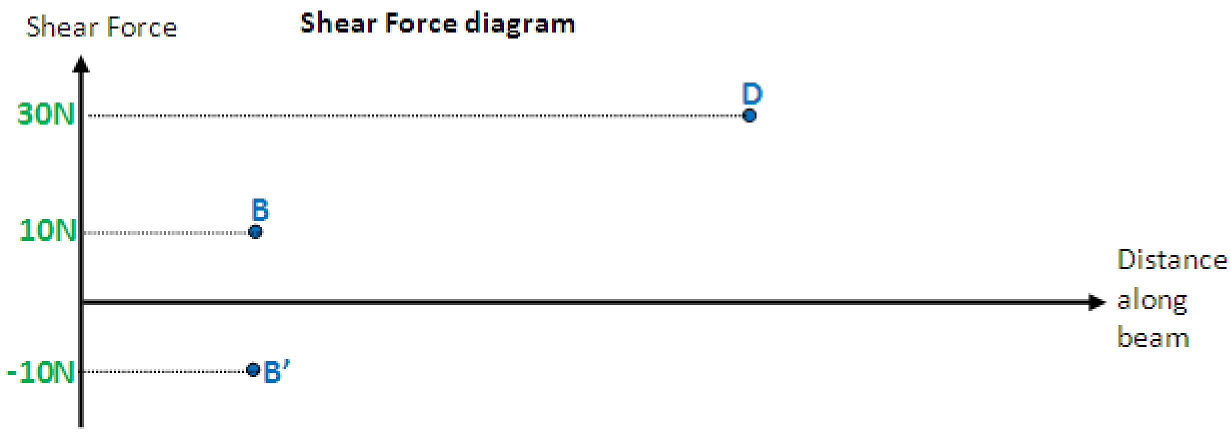
Shear force at B = 10N



Now place your paper JUST after point B:

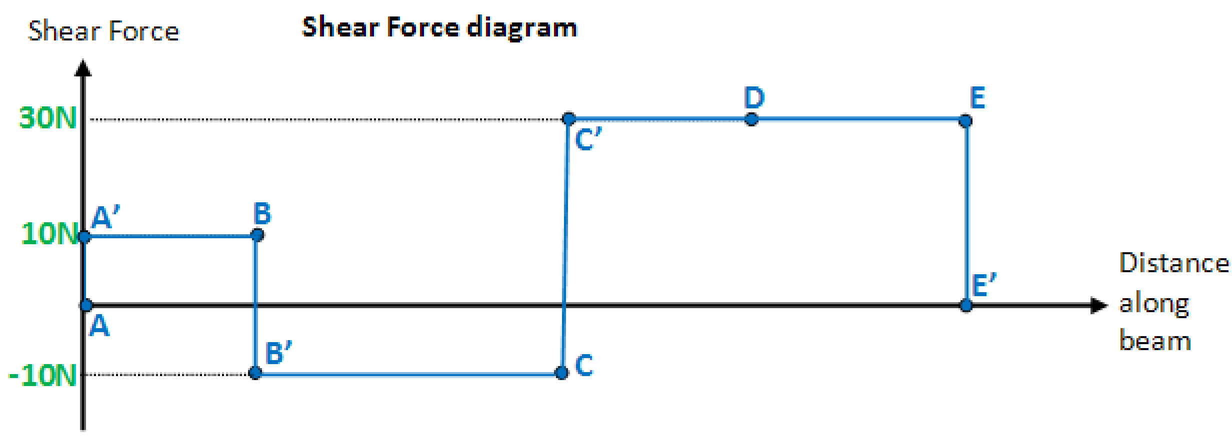


Shear force at B = 10N - 20N = -10N



(B' is vertically below B)

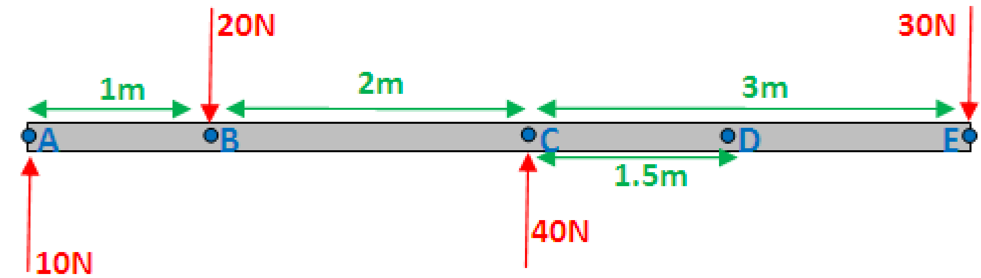
Now, do point A, D and E, and finally join the points. your diagram should look like the one below. If you don't understand why, leave a message on the discussion section of this page (its at the top), I will elaborate on the explanation:



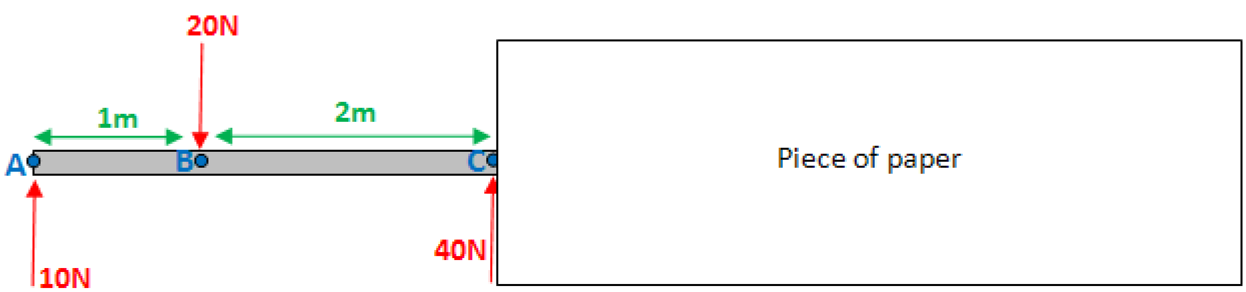
Notice how nothing exciting happens at point D, which is why you wouldn't normally analyse the shear force at that point. For clarity, when doing these diagrams it is recommended you move you paper from left to right, and hence analyse points A,B, C, and E, in that order. You can also do this procedure covering the left side instead of the right, your diagram will be "upside down" though. Both diagrams are correct.

Basic bending moment diagram

Bending moment refers to the internal moment that causes something to bend. When you bend a ruler, even though apply the forces/moments at the ends of the ruler, bending occurs all along the ruler, which indicates that there is a bending moment acting all along the ruler. Hence bending moment is shown on a bending moment diagram. The same case from before will be used here:

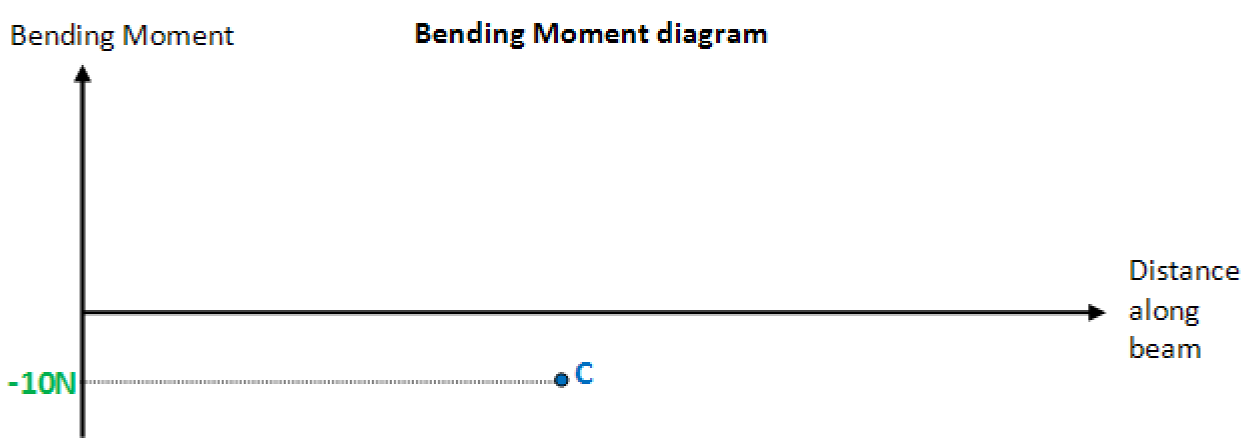


To work out the bending moment at any point, cover (with a piece of paper) everything to the right of that point, and take moments about that point. (I will take clockwise moments to be positive). To illustrate, I shall work out the bending moment at point C:



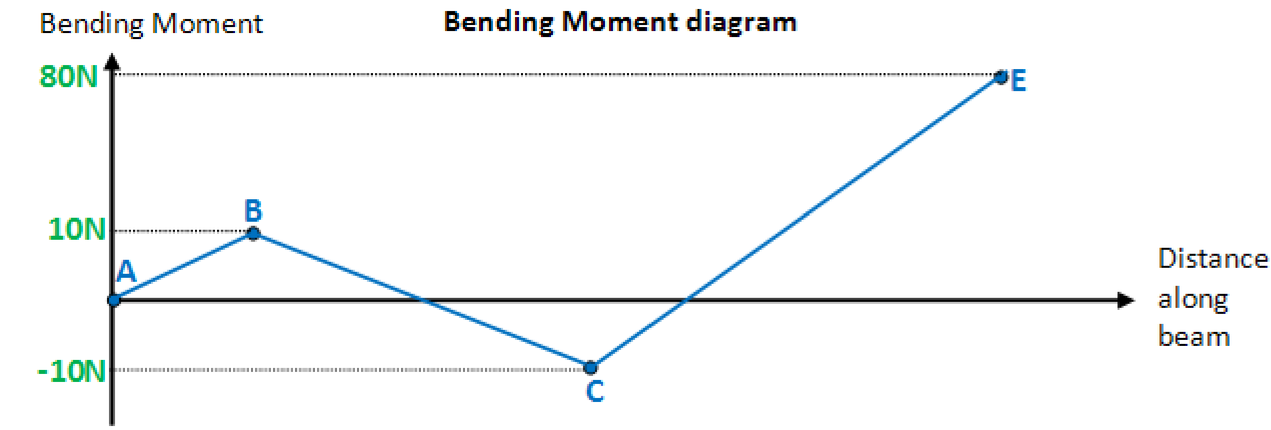
Bending moment at C = 10Nx3m - 20Nx2m = -10Nm

(Please note that the two diagrams below should show units in "Nm", not in "N" as it is currently showing)



Notice that there's no need to work out the bending moment "just before and just after" point C, (as in the case for the shear force diagram). This is because the 40N force at point C exerts no moment about point C, either way.

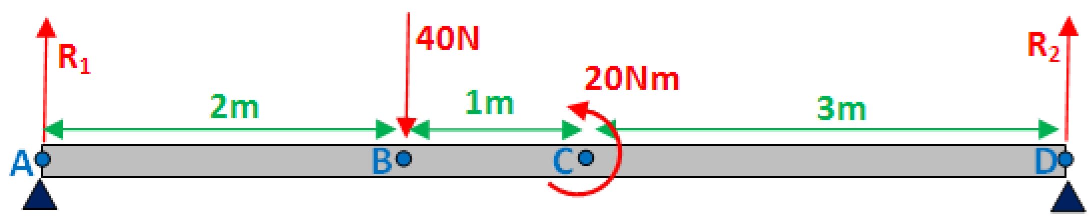
Repeating the procedure for points A,B and E, and joining all the points:



Normally you would expect the diagram to start and end at zero, in this case it doesn't. This is my fault, and it happened because I accidentally chose my forces such that there is a moment disequilibrium. i.e. take moments about any point (without covering the right of the point), and you'll notice that the moments aren't balanced, as they should be. It also means that if you're covering the left side as opposed to the right, you will get a completely different diagram. Sorry about this... Upon inspection, the forces are unbalanced, so it is immediately expected that the diagram will most likely not be balanced.

Point moments

Point moments are something that you may not have come across before. Below, a point moment of 20Nm is exerted at point C. Work out the reaction of A and D:



Force equilibrium: $R_1 + R_2 = 40$

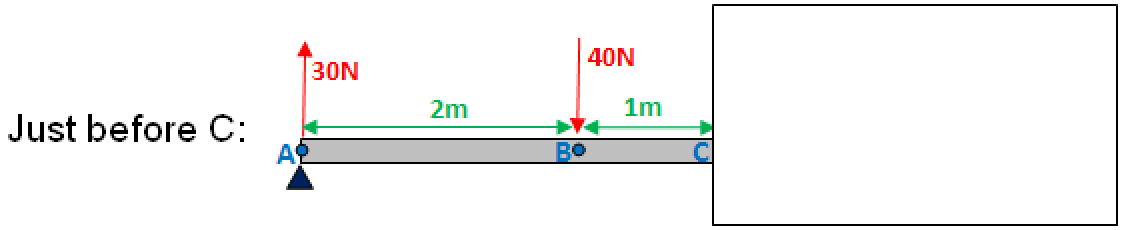
Taking moments about A (clockwise is positive): $40 \cdot 2 - 20 - 6 \cdot R_2 = 0$

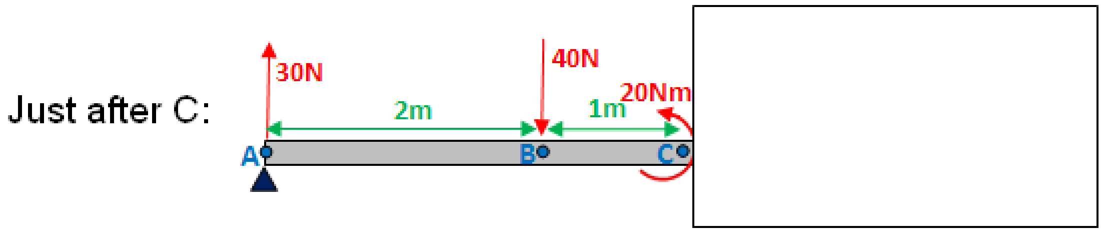
$R_1 = 30N$, $R_2 = 10N$

If instead you were to take moments about D you would get: $- 20 - 40 \cdot 4 + 6 \cdot R_1 = 0$

I think it's important for you to see that wherever you take moments about, the point moment is always taken as a negative (because it's a counter clockwise moment).

So how does a point moment affect the shear force and bending moment diagrams?
Well. It has absolutely no effect on the shear force diagram. You can just ignore point C when drawing the shear force diagram. When drawing the bending moment diagram you will need to work out the bending moment just before and just after point C:





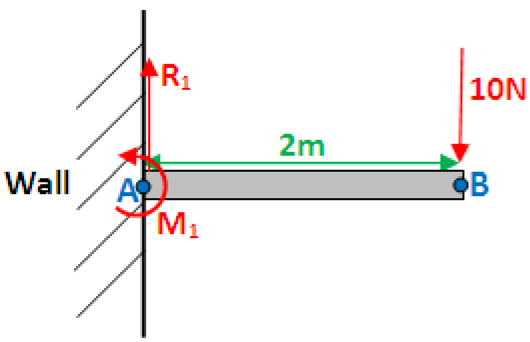
Just before: bending moment at C = $3 \cdot 30 - 1 \cdot 40 = 50$Nm

Just after: bending moment at C = $3 \cdot 30 - 1 \cdot 40 - 20 = 30$Nm

Then work out the bending moment at points A, B and D (no need to do before and after for these points). And plot.

Cantilever beam

Until now, you may have only dealt with "simply supported beams" (like in the diagram above), where a beam is supported by 2 pivots at either end. Below is a cantilever beam, which means - a beam that rigidly attached to a wall. Just like a pivot, the wall is capable of exerting an upwards reaction force R_1 on the beam. However it is also capable of exerting a point moment M_1 on the beam.



Force equilibrium: $R_1 = 10$N

Taking moments about A: $-M_1 + 10 \cdot 2 = 0 \rightarrow M_1 = 20$Nm

Uniformly Distributed Load (UDL)

Below is a brick lying on a beam. The weight of the brick is uniformly distributed on the beam (shown in diagram A). The brick has a weight of 5N per meter of brick (5N/m). Since the brick is 6 meters long the total weight of the brick is 30N. This is shown in diagram B. Diagram B is a simplification of diagram A. As you will see, you will need to be able to convert a type A diagram to a type B.

Diagram A:

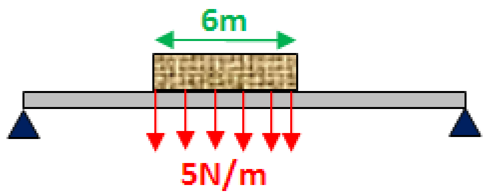
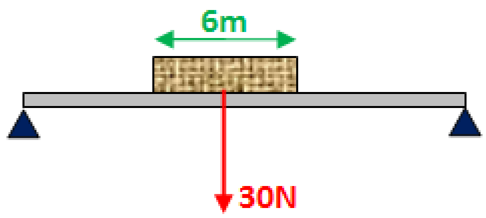
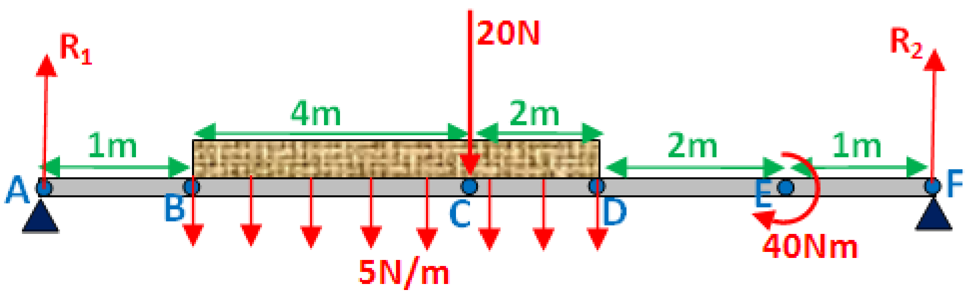


Diagram B:



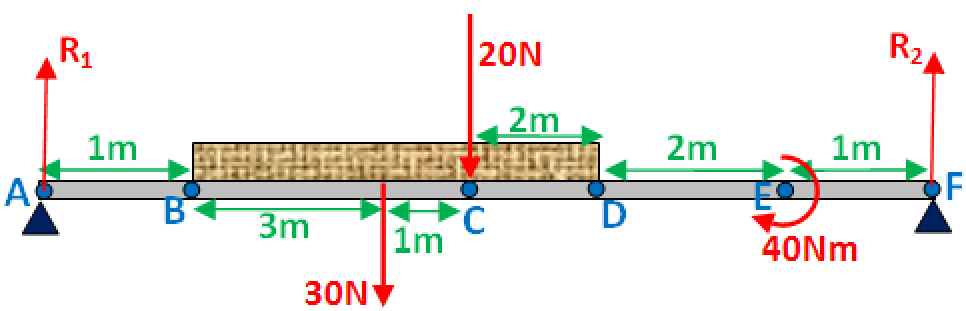
To make your life more difficult I have added an external force at point C, and a point moment to the diagram below. This is the most difficult type of question I can think of, and I will do the shear force and bending moment diagram for it, step by step.



Firstly identify the key points at which you will work out the shear force and bending moment at. These will be points: A,B,C,D,E and F.

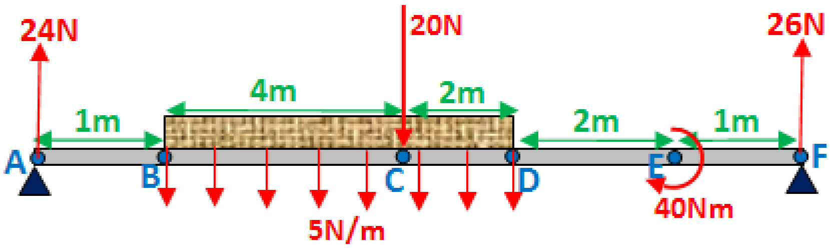
As you would have noticed when working out the bending moment and shear force at any given point, sometimes you just work it out at the point, and sometimes you work it out just before and after. Here is a summary: When drawing a shear force diagram, if you are dealing with a point force (points A,C and F in the above diagram), work out the shear force before and after the point. Otherwise (for points B and D), just work it out right at that point. When drawing a bending moment diagram, if you are dealing with a point moment (point E), work out the bending moment before and after the point. Otherwise (for points A,B,C,D, and F), work out the bending moment at the point.

After identifying the key points, you want to work out the values of R_1 and R_2. You now need to convert to a type B diagram, as shown below. Notice the 30N force acts right in the middle between points B and D.



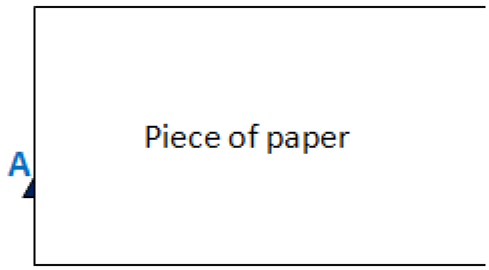
Force equilibrium: $R_1 + R_2 = 50$
Take moments about A: $4{\cdot}30 + 5{\cdot}20 + 40 - 10{\cdot}R_2 = 0$
$R_1 = 24N$, $R_2 = 26N$

Update original diagram:

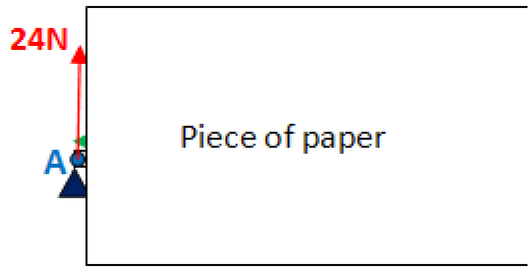


Shear force diagram

point A:

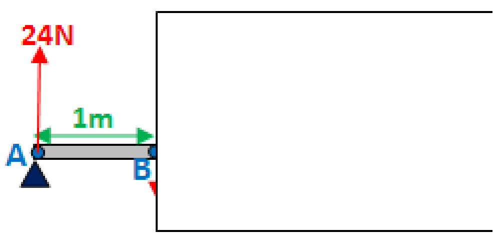


Before A – shear force = 0N



After A – shear force = 24N

point B:

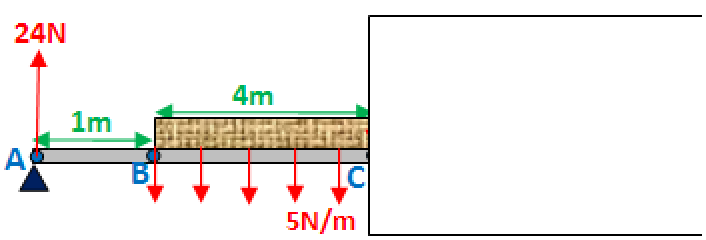


Shear force at B = 24N

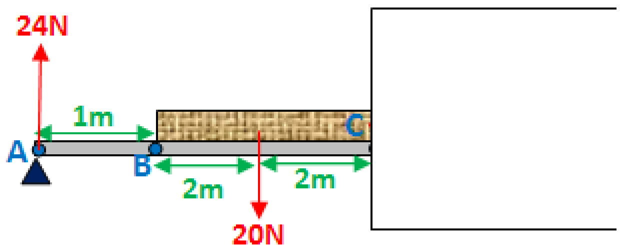
Notice that the uniformly distributed load has no effect on point B.

point C:

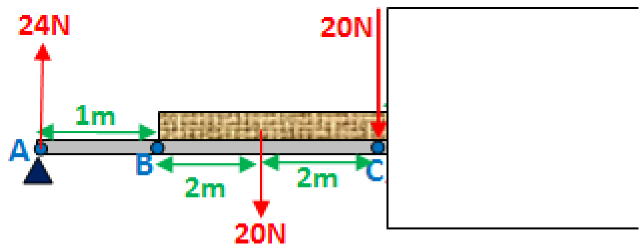
Just before C:



Now convert to a type B diagram. Total weight of brick from point B to C = 5x4 = 20N

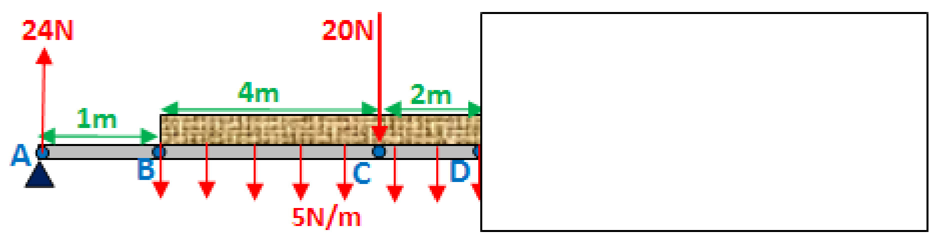


Shear force before C: 24 - 20 = 4N

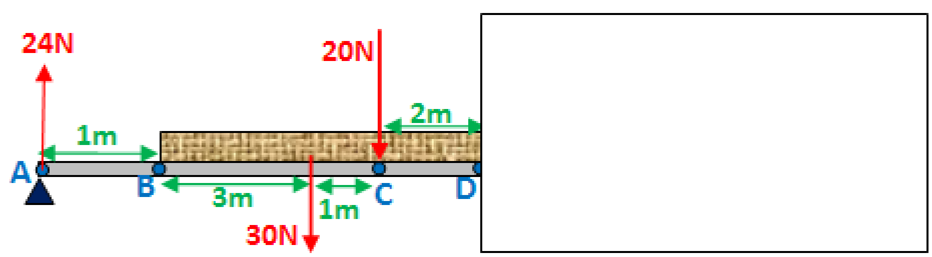


Shear force after C: 24 - 20 - 20 = -16N

point D:



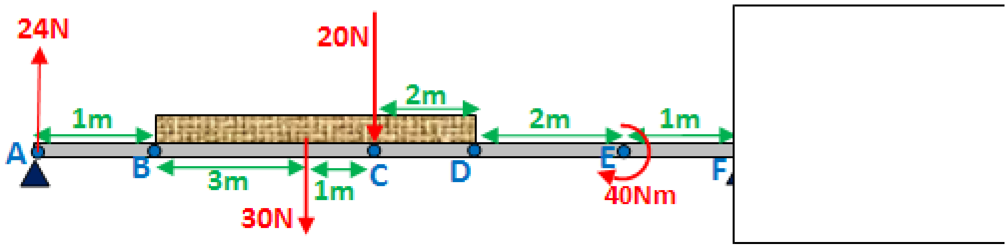
Convert to type B diagram:



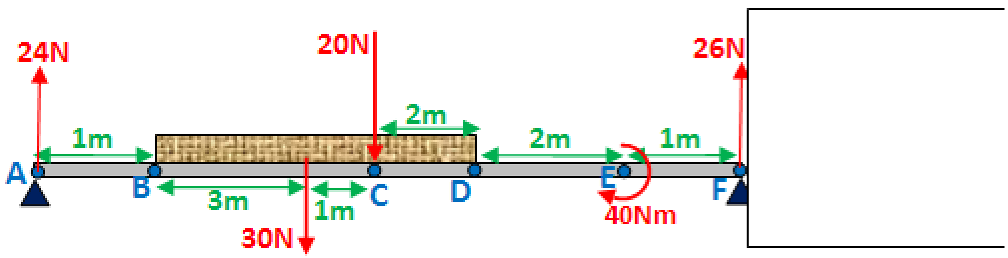
Shear force at D: 24 - 30 - 20 = -26N

point F:

(I have already converted to a type B diagram, below)

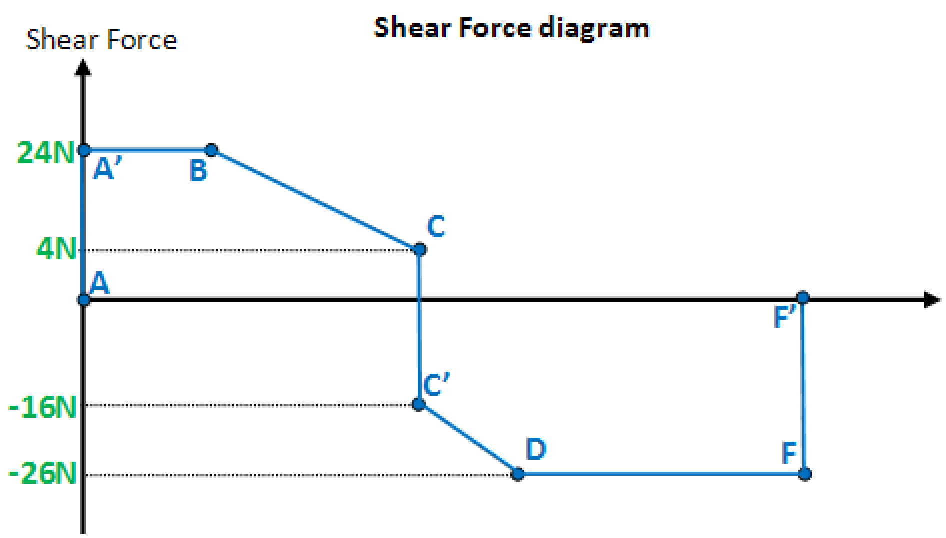


Shear force before F: 24 -30 -20 = -26N



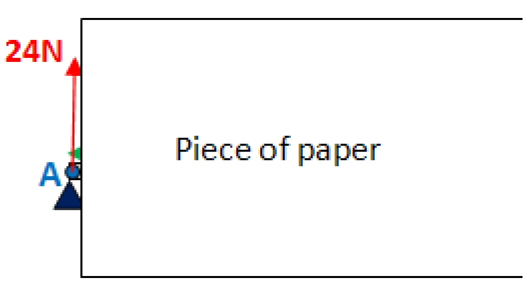
Shear force after F: 24 -30 -20 +26 = 0N

Finally plot all the points on the shear force diagram and join them up:



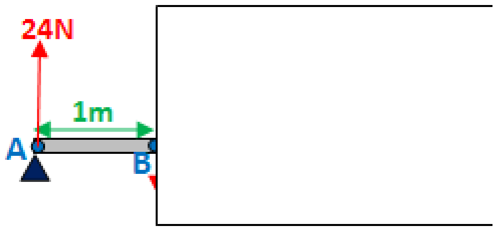
Bending moment diagram

Point A



Bending moment at A: 0Nm

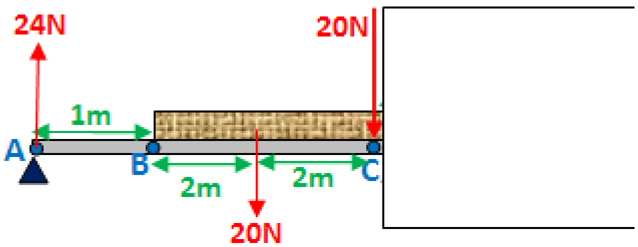
Point B



Bending moment at B: 24·1 = 24Nm

point C:

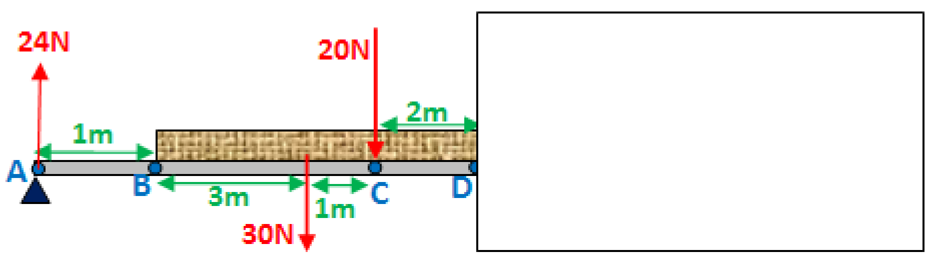
(I have already converted to a type B diagram, below)



Bending moment at C: 24·5 - 20·2 = 80Nm

point D:

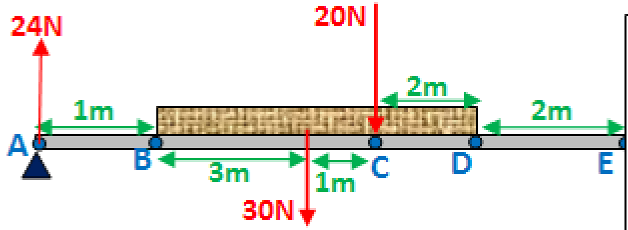
(I have already converted to a type B diagram, below)



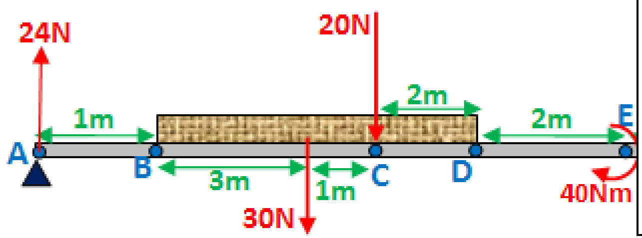
Bending moment at D: 24·7 - 30·3 - 20·2 = 38Nm

point E:

(I have already converted to a type B diagram, below)



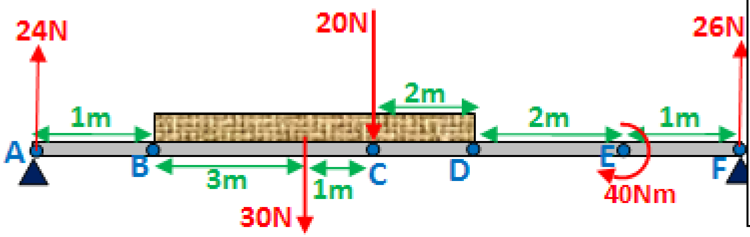
Bending moment just before E: 24·9 - 30·5 -20·4 = -14Nm



Bending moment just after E: 24·9 - 30·5 -20·4 + 40 = 26Nm

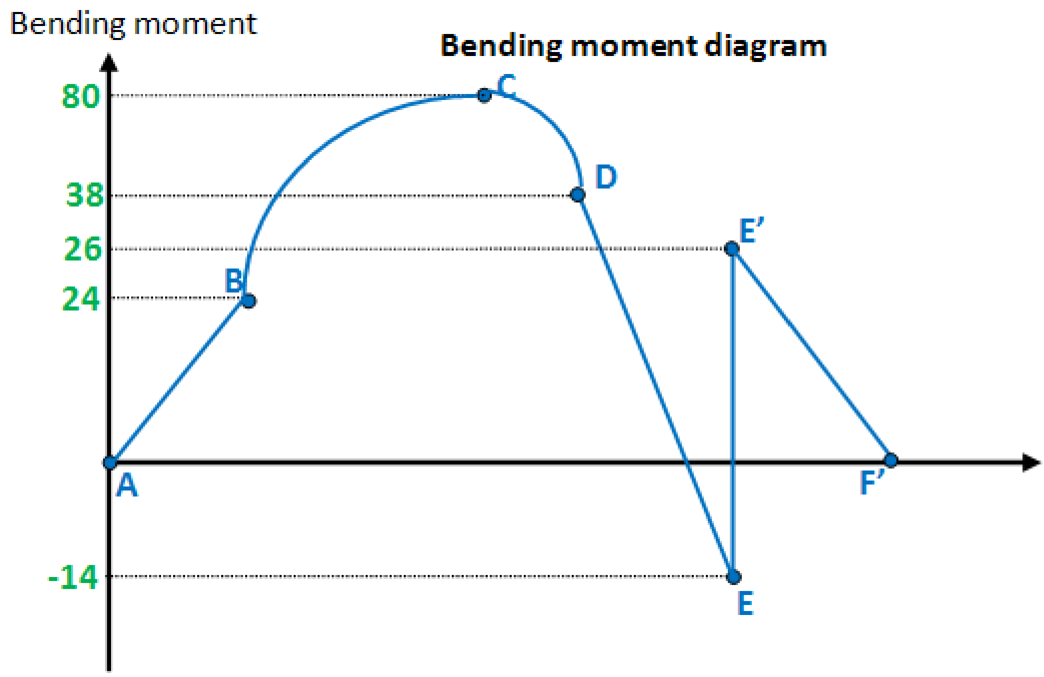
point F:

(I have already converted to a type B diagram, below)



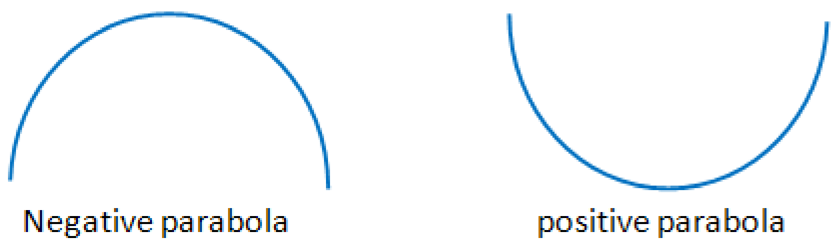
Bending moment at F: 24·10 - 30·6 - 20·5 + 40 = 0Nm

Finally, plot the points on the bending moment diagram. Join all the points up, EXCEPT those that are under the uniformly distributed load (UDL), which are points B,C and D. As seen below, you need to draw a curve between these points. Unless requested, I will not explain why this happens.



Note: The diagram is not at all drawn to scale.

I have drawn 2 curves. One from B to C, one from C to D. Notice that each of these curves resembles some part of a negative parabola.

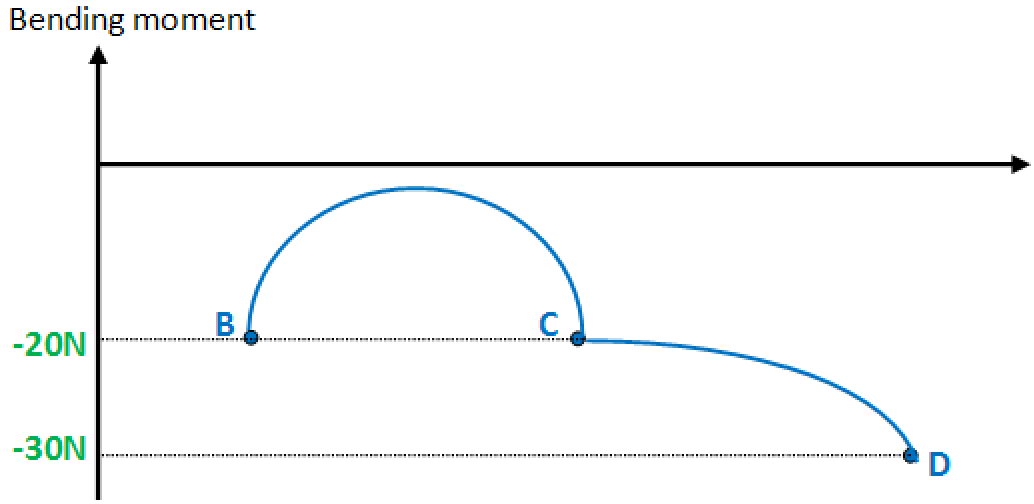


Rule: When drawing a bending moment diagram, under a UDL, you must connect the points with a curve. This curve must resemble some part of a negative parabola.

Note: The convention used throughout this page is "clockwise moments are taken as positive". If the convention was "counter-clockwise moments are taken as positive", you would need to draw a positive parabola.

Hypothetical scenario

For a hypothetical question, what if points B, C and D, were plotted as shown below. Notice how I have drawn the curves for this case.



If you wanted to find the peak of the curve, how would you do it? Simple. On the original diagram (used at the start of the question) add an additional point (point G), centrally between point B and C. Then work out the bending moment at point G.

That's it! If you have found this article useful, please comment in the discussion section (at the top of the page), as this will help me decide whether to write more articles like this. Also please comment if there are other topics you want covered, or you would like something in this article to be written more clearly.

IMPORTANT QUESTIONS
OBJECTIVE

1. 1. The bending moment for a certain portion of the beam is constant. For that section, shear force would be

- Zero
- Increasing
- Decreasing
- Constant

2. Hoop stress induced in a thin cylinder by winding it with wire under tension will be

- Compressive
- Tensile
- Shear
- Zero

3. What is the limiting value of Poisson's ratio?

- 0 and 0.5
- 1 and -0.5
- -1 and -0.5
- -1 and 0.5

4. Slenderness ratio has a dimension of

- m
- m^-1
- m^2
- Dimensionless quantity

5. Young's modulus of elasticity for a perfectly rigid body is

- Zero
- Unity
- Infinity
- None of these

6. Which material has the highest value of Poisson's ratio?

- Rubber
- Wood
- Copper
- Steel

7. Radial stress in a thin spherical pressure vessel is

- Equal to hoop stress
- Double the hoop stress
- Half the hoop stress
- Zero

8. The area under stress strain curve represents

- Breaking strength of material
- Toughness of material
- Hardness of material
- Energy required to cause failure

9. For a thin spherical shell subjected to internal pressure , the ratio of volumetric strain to diametrical circumferential strain is

- 1.25
- 1.5
- 2.0
- 3.0

10. Which of the following beam is likely to have the point of contraflexture?

- Cantilever beam

- Simply supported beam
- Beam with overhangs
- Beam fixed at both ends

11.The _____________ forces are used are used in the method of sections for the calculation of the internal forces.

a) Internal rotational
b) Couple rotational
c) Translational
d) External

12. Every point on the force vector which is the internal force is having the same magnitude and the same direction as the whole force vector have.

a) True

b) False

13. For getting the normal force on the supports, we do what?

a) Make the vertical sum of the forces equal to zero

b) Make the horizontal sum of the forces equal to zero

c) Make the moment sum of the forces equal to zero

d) Make the rotational sum of the forces equal to zero

14. For getting the horizontal component of the support reactions what do we do?

a) Make the vertical sum of the forces equal to zero

b) Make the horizontal sum of the forces equal to zero

c) Make the moment sum of the forces equal to zero

d) Make the rotational sum of the forces equal to zero

15. Twisting moment is also called as ___________

a) Moment of line

b) Moment of section

c) Moment of plane

d) Torsional moment

16. The loading generally act upon the ____________ of the body.

a) Centroid

b) Symmetrical centre

c) Rotational centre

d) Chiral centre

17. The area of does make the difference in the internal forces, that is if the area is large the internal force acting is also large and vice versa.

a) True

b) False

18. The magnitude of each loading will be ___________ at various points along the axis of the member of the beam.

a) Same

b) Different

c) Slightly different

d) Slightly same

19. Torsional moment is applied at the ___________ part of the beam.

a) The centroid

b) The left end

c) The right end

d) The axis beyond the body of the beam

20. Normal force is equal to _______________

a) The net horizontal force

b) The net vertical force with a negative sign

c) The net horizontal force with a negative sign

d) The net vertical force

21. If the normal force creates a tension then the force is said to be ____________

a) Positive

b) Negative

c) Rotational

d) Collinear

22. If the shear force creates a clockwise rotation then the force is said to be ____________

a) Positive

b) Negative

c) Rotational

d) Collinear

SHORT QUESTIONS

1. Define Elastic Materials ?
2. Define Plastic Materials ?
3. Define Brittle Materials ?
4. Define ductile Materials ?
5. Give Any Two Example Of Ductile Materials ?
6. Give Any Two Example Of Brittle Materials ?
7. Describe tensile test?
8. Rigid body?
9. Describe compresion test?
10. Bulk modulus?
11. FOS
12. define load?
13. Define tensile load ?
14. Define compressive load ?
15. Define shear stress load ?
16. define stress?
17. define strain?
18. Define hooke's law?
19. define volumetric strain ?
20. unit of stress, strain. Modulus of rigidity?
21. define bending moment?
22. Define shear force?
23. define types of load?
24. define types of beam?
25. define types of supports ?
26. define types of reactions ?
27. define point of contraflexure?
28. define elastisity ?
29. relation between modulus canstant?
30. define bulk modulus ?

LONG QUESTIONS

1. Describe compresion test?
2. Describe tensile test?
3. discribe stress-strain digram/graph ?
4. define hooke's law?
5. derivate the formula of elongation of bars?
6. Describe E,C,K. Relationship also?
7. Define mechanical properties of materials ?
8. defferentiate between load and stress?
9. describe sfd &bmd and point of contraflexure.?